Literature Review of Multifactor Models and Recent Trends

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Abstract: Numerous papers about multifactor models are proposed to explain the cross-section of expected returns. This paper provides an overview of existing multifactor models, particularly the Fama-French three-factor model and the five-factor model. Respectively, factor establishments, empirical strategy and results and comparison between two models will be fully included. The paper also critically discusses the deficiencies and provides potential improvements for these traditional models. Innovatively, this paper suggests that future research should concentrate more on the modifications of empirical tests on these models and, apart from that, potential factors for different kinds of securities require further exploration as well.

1. Introduction

The Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), and Black (1972) has been regarded as the perfect method to explicate the risk factors and average returns for a long time. In this model, market beta is recognized as a single paramount factor to explain the cross-section expected returns. However, β alone actually cannot explain the cross-section of average returns during the 1963-1990 sample period (Fama and French, 1992). Models such as the three-factor model (Fama and French, 1993) and the five-factor model (Fama and French, 2015a) have been proposed to better explain the expected returns. However, these models also receive critiques and cannot explain many anomalies. Such models still have a long way to go. Therefore, it is meaningful to retrospect the development of significant models and bring more insight into future trends.

The size effect, which cannot be explained by the CAPM, is the first anomaly proposed (Banz, 1981). It is believed that ME (the market equity, a stock's price times shares outstanding) should also be included in the explanatory model of the cross-section of average returns. Another anomaly is the E/P (earnings-price ratios) which could explain part of the average stock returns in tests when combined with size (ME) and market (β) (Basu, 1983). Moreover, leverage (another indicator of risk) should be included in the tests combining ME and β in order to explain much of the expected returns (Bhandari, 1988), and BE/ME (book-to-market equity) also plays role in explaining the returns (Chan et al, 1991).

Researchers have discovered more about the factors or patterns used to explain average stock returns. Multifactor models of stock returns are topical areas to explore in recent years. Continuously, there are many related models proposed by different people to demonstrate the relationship between various factors and stock returns. With so many factors that seem should be included in one explanation model, it is worthy to find a paramount factor model that can have the same explanatory power and abandon redundant factors. For example, BE/ME, ME, leverage, and E/P are all scaled versions of price. It is rational to think that some of them own the same explanatory power for average return (Fama and French, 1992). Fama and French (1993) carry out the research on 5 common risk factors related to the returns on stocks and bonds. They state that the three-factor model which includes market, size and BE/ME does a great job in explaining average stock returns and, additionally, two term-structure factors- *TERM* (a proxy for an unexpected change in interest rates) and *DEF* (proxy for default factor)- capture most of the variation in bond returns. Further, Fama and French (1996) continuously add more explanations to enhance the three-factor model and meanwhile propose some troublesome anomalies.

However, this is not the whole picture. Researchers are more likely to wonder whether there are other necessary factors that should be included in the tests in order to strengthen the explanatory power of the model. The five-factor model (Fama and French, 2015a), including size, market, BE/ME, profitability and investment, are proved to perform better than the previous three-factor model. However, nothing is perfect. The problem of this model is that profitability and investment factors seem to blanket the role of the *HML* (high minus low, a proxy for BE/ME) factor for explaining average returns (Fama and French, 2015a). Besides, the five-factor model performs badly in explaining the anomalies in accruals and momentum. And most of the unexplained returns are presented in microcaps because the negative exposures to *RMW* (robust minus weak, a proxy for profitability) and *CMA* (conservative minus aggressive, a proxy for investment) cannot be totally explained by the five-factor model (Fama and French, 2016).

The above call us into question. On the one hand, it is more likely that we exclude some of the important factors that play a key role in explaining average returns, probably in the future, we establish momentum factor and accrual factor to explain momentum effect and accrual anomalies, respectively. On the other hand, there is already existing research conducted by others to further propose risk factors to explain the short-term persistence in mutual fund returns (Carhart, 1997). More interestingly, Daniel and Titman (1997) seem to object to the Fama and French factor model, since they argue that it is characteristics rather than risk factors that determine expected returns. Despite the fact that the factor models indeed enhance the explanation, the empirical methods used in the tests can be improved. For instance, testing other portfolios, measuring the regression intercepts and slopes more precisely and presenting confidence intervals for specific statistics could be selected in the tests (Lewellen et al, 2010). In contrast, Hou, Xue, and Zhang (2015) confirm that their q-factor model, including market, size, investment, and profitability, outperforms the Fama-French models in capturing some of the anomalies. Apart from all of the various models, we get a better understanding of multiple factors used to explain the same thing. However, the problem of which model stands out still remains. Thereby, the identification of the best combination of factors based on previous research comes into view (Barillas and Shanken, 2015). A Bayesian procedure to evaluate the best model with the highest probability has been used, which shows that a six-factor model, including market, investment, profitability, size, value and momentum factor would be the best. Certainly, the exploration will be constantly conducted and improved with great efforts in future research.

In the next few sections, we focus on the factor establishments, the regression details, the detailed explanations and anomalies, comparison between different models and critical evaluation. Section II explains the three-factor model and the five-factor model. Moreover, Section III critically discusses the judgments and modifications of model testing approaches. Lastly, Section IV concludes the major contributions of the above multifactor models and proposes innovative or prospecting ideas for future research in asset-pricing models.

2. Fama and French Multifactor Models of Capital Asset Pricing

2.1. Factor Establishments of Each Model

The three-factor model (Fama and French, 1993), contradicting the assumption that β alone could explain the cross-section of average returns, is first proposed to enrich CAPM. This model uses stock data trading on NYSE, Amex, and NASDAQ with stocking price data from CRSP and accounting data from COMPUSTAT. (Fama and French, 1992).

Briefly, the three risk factors are market, size, and book-to-market equity. In regression tests, the excess return on the market portfolio of stocks $(R_M - R_F)$ is used to mimic the market factor. Also, the proxy for the size factor is *SMB* (small minus big). *HML* could mimic book-to-market equity.

Along with more diversified development, there is much evidence proving that profitability and investment, another two risk factors, should also be added into the previous model (Fama and French, 2015a). The proxy for profitability is RMW_t. CMA_t represents the investment factor. This is due to the evidence that higher expected growth in book equity ($dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$), i.e. investment,– implies a lower expected return (r) and higher expected earnings ($Y_{t+\tau}$) generate a higher expected return when holding other variables constant:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{1}$$

In addition, the excess returns on different sorts of portfolios, denoted by $Rt - RF(t)$, are treated as dependent variables in the regressions. The detailed explanations about each factor are illustrated in Table 1.

The factor construction of the three-factor model mainly uses 2×3 and 5×5 sorts. To investigate if there are different patterns in factor definitions, the five-factor model uses various sorts, such as 2×2 sorts, 2×3 sorts, 2×2×2×2 sorts (Fama and French, 2015a). In the 2×3 sorts and 2×2 sorts, the second part of these sortings describes the BE/ME group, the OP (profitability) group, or the Inv (investment) group. The size factor always uses the median NYSE size breakpoint to divide stocks into 2 groups. As for value, profitability and investment factors, either stocks are equally divided into 2 groups, or they are divided into 3 groups by using $30^{\text{th}}$and 70^{th} breakpoints. As for 2×2×2×2 sorts, the factors use 16 value-weighted portfolios to isolate the effect of each factor. All four factors use NYSE median as breakpoints to divide stocks into 2 groups respectively. Similarly, the 5×5 sorts allocate stocks into 5 size quintiles and 5 book-to-market quintiles individually according to NYSE breakpoints (Fama and French, 1993). The intersection of 25 portfolios is created. Finally turns out, different methods of factor variation do not affect too much on model performance. But the 2×2 factors are indeed best qualified for the five-factor model.

2.2. Factor Model Empirical Strategy and Results

Fama and French (1993) test the combined effect of the three factors, i.e. $R_M - R_F$, *SMB*, and *HML* on average returns. The regression for the three-factor model is given as:

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t) \tag{2}$$

Not surprisingly, the three factors capture the greater variation in the returns. Compared with the regression tests that use market factor or both size and value factor, R^2 increases apparently. For example, if the regression only uses market factor, R^2 is not as high as expected in terms of small and high BE/ME portfolios, approximately 70% or less. Large size with higher BE/ME possibly cannot be perfectly explained by *SMB* and *HML*. After combining them all, all of the value of R^2 is 0.93 on average.

Progressively, by comprising other two variables into the three-factor model, (Fama and French, 2015a)

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} \tag{3}$$

the five-factor model produces lower GRS statistics, smaller intercepts, and higher R^2. GRS statistics (Gibbons, Ross, and Shanken, 1989) are used to examine the model efficiency by testing whether the intercepts are zero. The lower the GRS statistic is, the more likely the intercepts will be zero, the more variation the model could explain. The five-factor model intercepts are the minimum, only leaving 42–54% of the variation in average returns unexplained, compared to CAPM with intercept variation ranging from about 1.26 to 1.55 and the three-factor model with the intercept variations ranging from 54% to 68%. More intriguingly, dropping the *HML* factor in the four-factor model which actually performs as good as the five-factor model being discussed. Actually, the average return of the *HML* factor is largely absorbed by *RMW* and *CMA*, so *HML* is redundant.

Table 1: Factors used in three-factor model and five-factor model.

<table>
<tr><th></th><th>Three-factor model</th><th>Five-factor model</th></tr>
<tr><td rowspan="5">Proxies used in regression tests</td><td colspan="2">Market β: RM - RF
RM: the return on the value-weighted portfolio of the stocks in the size-BE/ME portfolios
RF: one-month bill rate</td></tr>
<tr><td colspan="2">Size factor: SMB (Small minus Big)
SMB: the returns of small portfolios (S) - the returns of big portfolios (hold BE/ME constant) (B)
S and B: the median NYSE breakpoint is used to divide stocks into two groups, small and big.</td></tr>
<tr><td colspan="2">Value factor (BE/ME): HML (High minus Low)
HML: the returns of high BE/ME portfolios (H) - the returns of low BE/ME portfolios (L), holding size constant
H and L: the stocks are divided into three groups by using 30^{th} and 70^{th} breakpoints.</td></tr>
<tr><td rowspan="2"></td><td>Profitability factor: RMW
RMW: the returns of stocks with robust earnings (R) - the returns of stocks with weak earnings (W), holding size constant in 2×2 sorts
R and W: the median NYSE breakpoint is used to divide stocks into two groups, robust and weak.</td></tr>
<tr><td>Investment factor: CMA
CMA: the returns of stocks with conservative investments (C)- the returns of stocks with aggressive investments (A), holding size constant in 2×2 sorts.
C and A: the median NYSE breakpoint is used to divide stocks into two groups, conservative and aggressive.</td></tr>
</table>

2.3. Comparison of the Performances between Two Models

Compared to CAPM, the three-factor model could explain the returns of portfolios formed on E/P, C/P (cash flow-price ratio), and sales growth. For example, companies that have high sales growth, low C/P and E/P tend to load negatively on the *HML* factor. They are the signs of strong firms. The reversal of long-term returns (DeBondt and Thaler, 1985) is also captured by the three-factor model. Literally, stocks that earn low long-term past returns are more likely to load positively on *HML* and *SMB* slopes, i.e., these smaller-size stocks seem to be long-term losers and kind of distressed. But they are expected to earn higher future average returns due to long-term reversal. Specifically, according to significant level tests, t-statistics on *SMB* slopes are greater than 10. The *SMB* is negatively related to returns. The *HML* is positively related to returns. Interestingly, the participation of both *SMB* and *HML* reduces the slope for the market factor from 1.4 to about 1 due to the correlation among *RM(t)-RF(t), SMB* and *HML*.

However, the three-factor model still lacks the reasoning for the continuation of short-term returns (Jegadeesh and Titman, 1993). Although the so-called short-term losers which are stocks with lower short-term past returns also have positive *HML* slopes, they still gain worse future earnings. Short-term winners loading negatively on *HML* are still the winners in the near future. As for double-sort portfolios, average returns have a reverse effect only when the portfolios are formed on returns of the past 4 years prior to the final formation. Instead, if they use returns of the past 12 months, the continuation effect takes the dominant position. Strangely enough, when forming portfolios based on returns of all past 5 years, including the year of formation, the short-term continuation effect is large enough to offset the long-term reversal effect.

Besides, the three-factor model performs poorly when explaining the returns of portfolios sorted on profitability and investment. However, the five-factor model, an extended version of the three-factor model, could explain more of the variation in stock average returns. Since profitability (*RMW*) and investment (*CMA*) factors are considered, the intercepts of Size-OP-Inv portfolios are absolutely improved. Furthermore, the dispersion of unexplained average returns for the five-factor model is relatively lower than the three-factor model. The five-factor model intercepts leave 42–54% of the variation unexplained, compared to the three-factor model with the intercept variations ranging from 54% to 68%.

The anomalies which cannot be captured by the five-factor model still exist. Small stocks, which have negative slopes on *RMW* and *CMA*, have low average returns (Fama and French, 2016). The patterns of their returns are similar to the stocks which invest a lot but earn less. Big stocks, however, have positive returns even though they also invest a lot and earn less. Negative *CMA* means investing a lot. Negative *RMW* does not always mean low profitability. Therefore, unexpected high investment for small stocks is still strange, though.

Scratch a little deeper, the phenomenon of small stocks could be beneficial for testing other possible factors, such as market β, net share issues, volatility, accruals, and momentum (Fama and French, 2016). CAPM predicts that there exists a relationship between β and average returns. Actually, it is not like that. GRS tests reject CAPM in the β sorts. Intercepts also change a lot with size and β in the CAPM and three-factor model. Conversely, the five-factor model and four-factor model which excludes *HML* show the least dispersion of unexplained returns. By conducting the tests, it is proved that the slopes of *RMW* and *CMA* factors could offset the predictive effect (about average returns) of market β and size.

In particular, the five-factor model captures the most variation in average returns in the Size-NI (net issues) portfolios. The model absorbs repurchase anomaly. However, the intercepts in higher NI portfolios are negative. The firms that issue new shares, combined with low profitability and high investment, are less likely to be explained by the five-factor model (Fama and French, 2016).

As for volatility, the model reduces the intercepts to nearly zero by adding *RMW* and *CMA* as well (Fama and French, 2016). But the lethal problem remains. The low returns of small size and high volatility portfolios with negative slopes on *RMW* and *CMA* still cannot be understood.

Then come two of the trickiest anomalies. Firstly, the five-factor model seems to perform worse in explaining Size-AC (Accrual) portfolios. Adding *RMW* into the existing four-factor model increases the intercepts of the Size-AC stocks. Even though including the *CMA* factor, the intercept problems still exist in the small size and high investment portfolio (Fama and French, 2016). Apart from that, the short-term continuation problem in the three-factor model actually remains unsolved in the five-factor model. All models which do not consist of momentum factor (*MOM*) explain less about variation in average returns of Size-Prior 2-12 portfolios (Momentum). Adding the *MOM* factor to construct a six-factor model decreases intercepts a little bit. However, the returns of small size portfolios which illustrate strong momentum effects are still left unexplained by the six-factor model (Fama and French, 2016). Overall, the summary of the anomalies that cannot be captured by two models is presented in Table 2.

Table 2: Unexplained anomalies of the two models.

<table>
<tr><th></th><th>Three-factor model</th><th>Five-factor model</th></tr>
<tr><td rowspan="5">Anomalies unexplained</td><td>The returns of portfolios sorted on profitability and investment</td><td>The low returns of small stocks with negative slopes on RMW and CMA</td></tr>
<tr><td colspan="2">Short-term continuation effect (MOM factor)</td></tr>
<tr><td rowspan="3"></td><td>The low returns of small size and high volatility portfolios with negative slopes on RMW and CMA</td></tr>
<tr><td>The unexplained returns of Size-AC (Accrual) portfolios</td></tr>
<tr><td>The negative intercepts in high NI portfolios</td></tr>
</table>

3. Critical Judgements about Multifactor Models

The critique will mainly highlight two parts. Specifically, whether the *HML* factor is redundant or not requires discussion. What's more, improvements for testing different models are mentioned as well.

Fundamentally, from the perspective of the FF five-factor model itself, it performs as good as the four-factor model which excludes the *HML* factor. It is known from the regression tests that the average return of *HML* could be basically absorbed by other factors. *HML* factor seems to be redundant in the five-factor model when explaining the variation in average returns (Fama and French, 2015a). Additionally, a six-factor model that comprises the *MOM* factor leaves the least unexplained average returns compared with all other models (Fama and French, 2016). Moreover, another q-factor model (Hou et al, 2015) which only includes market, size, investment and profitability factors is proved to be better than the FF three-factor model, since more anomalies could now be explained. The reason why this model does not have *HML* and momentum factors is that these two factors could be inaccurate to be included (Hou et al, 2015). And also, HXZ uses more timely versions of profitability and investment factors.

However, all models are simply idealized versions of reality, therefore, they are not comprehensive. On the one hand, the factors lack accuracy because they are only proxies for real risk factors. On the other hand, using large p-value alone to determine the efficiency of previous models is more likely to be imprecise. To solve this dilemma, it has come up with a more accurate method which is the Bayesian procedure (Barillas and Shanken, 2015) to compute exact model probabilities in order to form the best combination of factors among all possible alternatives. The final results show that the six-factor model *{Mkt (market) IA (investment) ROE (profitability) SMB (size)* HML^m *(value) UMD (momentum)}* wins with the highest probability. Compared with the q-factor model and FF five-factor model, the value factor should be considered in this way. HML^m is constructed based on the latest book-to-market rankings. A more timely value factor could lead to more accurate final results. Furthermore, the top-ranking models which perform better than the FF five-factor model and the q-factor model all have *UMD* in common.

The measures for improving the regression tests need to be reviewed. It does not seem so convincing to explain the BE/ME and size effects because of common ground among previous multifactor models (Lewellen et al, 2010). Traditionally, high R^2 are thought to represent a well-performing model. However, it may not be the truth, because the sampling issue could sometimes matter. More importantly, the covariance structure could exist between different factors, i.e., the high correlation between the factor and the common variation in returns, and that would also result in a higher R^2. To address this issue, including other industry-sorted portfolios will help broaden the range. Also, emphasize more on theoretical restrictions on the regression intercepts and slopes instead of ignoring them. Apart from that, reporting GLS R^2 rather than OLS R^2 could be more stable and reliable. The final thing that helps mitigate the sampling problem is to use confidence intervals for cross-sectional R^2 or other important statistics.

4. Conclusion

This paper puts emphasis on the development, analysis and, critiques of multifactor models mainly proposed by Fama and French. Initially, the paper describes the general background, reasons, aims, and results of these models. And then further details about the factor establishments, regression tests, direct evidence and explanations, and comparison of performances among these models are fully discussed respectively. The basic conclusion is that the five-factor model indeed performs better than the three-factor model. Although they contribute more to the explanation of average stock returns than the original CAPM does, they still leave some of the anomalies unexplained, for example, they cannot explain too much about the portfolios of small size, high BE/ME, low profitability but the high investment. As for FF5, the *HML* factor seems to be redundant and the *MOM* factor, however, could improve the explanation.

The paper also offers critiques about the previous traditional models. For example, a timely *HML* factor is no longer redundant, at least when using a Bayesian procedure to compute model probabilities. The problems of sampling and covariance structures in traditional empirical tests could be solved by sorting diversified portfolios and using stable statistics.

Although, exploring the size and book-to-market effects on average returns could expose a certain degree of the nature behind these economic risks (Fama and French, 1992). That is, the distressed firms behave more sensitively to economic conditions, causing distress factors to price expected returns (Chan et al, 1991). This paper only presents a limited overview of the previous classical models of asset-pricing. Future research still has a long way to go. First and foremost, whether the factors in the multifactor models expose the same patterns of stock average returns in historical periods or future periods requires further discussion. Additionally, the momentum factor closely associated with investors' reaction to the market could be another direction of multifactor model research. In order to solve the separation of the asset-pricing model and the macro-economy, the combination of macro-economy and micro-enterprise probably brings about undefined factors for the explanation of average returns. Apart from that, other kinds of capital markets in different regions probably need different sets of factors to capture not only stock average returns but also returns of other securities like mutual bonds, etc.

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